

Inference of Global Progress Properties for Dynamically Interleaved Multiparty Sessions

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The problem

$$a(y).b(z).y?(x).z!\langle x \rangle$$
$$\bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle$$

- two distinct sessions
- each session is well typed
- the system gets stuck

The problem

$a(y).b(z).y?(x).z!\langle x \rangle$ $y : ?int$ $z : !int$

$\bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle$ $y : !int$ $z : ?int$

- two distinct sessions
- each session is well typed
- the system gets stuck

The “interaction” type system

If $\vdash P$, then P never gets stuck

☺ Bettini, Coppo, D’Antoni, De Luca, Dezani-Ciancaglini, Yoshida, **Global Progress in Dynamically Interleaved Multiparty Sessions**, CONCUR 2008

☹ **not syntax-directed**

Outline

- ① Progress
- ② Key ideas of the type system
- ③ Two examples
- ④ Remarks

Progress 1/2

If $P \rightarrow^* \mathcal{E} [s?(x).P']$
Then $\rightarrow^* \mathcal{E}' [s?(x).P' \mid s : m \cdot h]$

If $P \rightarrow^* \mathcal{E} [s : m \cdot h]$
Then $\rightarrow^* \mathcal{E}' [s : m \cdot h \mid s?(x).P']$

A process without progress

$$a(y).b(z).y?(x).z!\langle x \rangle \mid \bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle$$

A process without progress

$$a(y).b(z).y?(x).z!\langle x \rangle \mid \bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle$$

\downarrow_*

$$(\nu s)(\nu s')(s?(x).s'!\langle x \rangle \mid s'?(x).s!\langle c \rangle \mid s : \emptyset \mid s' : \emptyset)$$

A process without progress

$$a(y).b(z).y?(x).z!\langle x \rangle \mid \bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle$$
$$\downarrow_*$$
$$(\nu s)(\nu s')(s?(x).s'!\langle x \rangle \mid s'?(x).s!\langle c \rangle \mid s : \emptyset \mid s' : \emptyset)$$

Progress 2/2

A **good** process that looks like a **bad** one

$$P \rightarrow^* \mathcal{E}[s?(x).P' \mid \bar{b}(y).s!\langle 3 \rangle.Q']$$

A **bad** process that looks like a **good** one

$c(y).$ (a process that gets stuck)

Progress 2/2

A **good** process that looks like a **bad** one

$$P \rightarrow^* \mathcal{E} [s?(x).P' \mid \bar{b}(y).s!\langle 3 \rangle.Q']$$

A **bad** process that looks like a **good** one

$c(y).(a \text{ process that gets stuck})$

Idea

- define progress modulo **catalyzers**
- catalyzer = missing participant that never gets stuck

Consequence

- session initiation can be considered **non-blocking**

Interaction type system: basic ideas

- 1 associate processes with **dependencies** $a < b$

“an action of service a blocks an action of service b ”

- 2 a process is well typed if it yields **no circular dependencies**

Computing service dependencies

$$a(y).b(z).y?(x).z!\langle x \rangle \quad a < b$$

$$\bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle \quad b < a$$

Service names as messages

$$a(y).b(z).y?(x).z!\langle x \rangle \quad a < b$$

$$\bar{c}(t).t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle$$

$$c(t).t!\langle a \rangle$$

Service names as messages

$a(y).b(z).y?(x).z!\langle x \rangle$ $a < b$

$t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle$

$t!\langle a \rangle$



Service names as messages

$a(y).b(z).y?(x).z!\langle x \rangle$ $a < b$

$\bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle$ $b < a$

Service names as messages

$a(y).b(z).y?(x).z!\langle x \rangle$ $a < b$

$\bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle$

Idea

- identify a class of safe services even if mutually dependent
- restrict messages to services in this class

Nested services

Definition

a is a **nested service** if $\lambda < a$ implies that λ is a nested service

$\bar{a}(y).\bar{a}(z).z?(x).y?(x')$

$a < a$



$\bar{a}(y).\bar{b}(z).z?(x).y?(x')$
 $| \bar{b}(z).\bar{a}(y).y?(x).z?(x')$

$b < a$



$a < b$

$\bar{a}(y).\bar{b}(z).y?(x).z?(x')$

$y < b$



Nested?

Private services

$$a(y).(vb)(b(z).z?(x).y!\langle x \rangle)$$

- no catalyzer can help starting the session on b

Private services

$$a(y).(\nu b)(b(z).z?(x).y!\langle x \rangle)$$

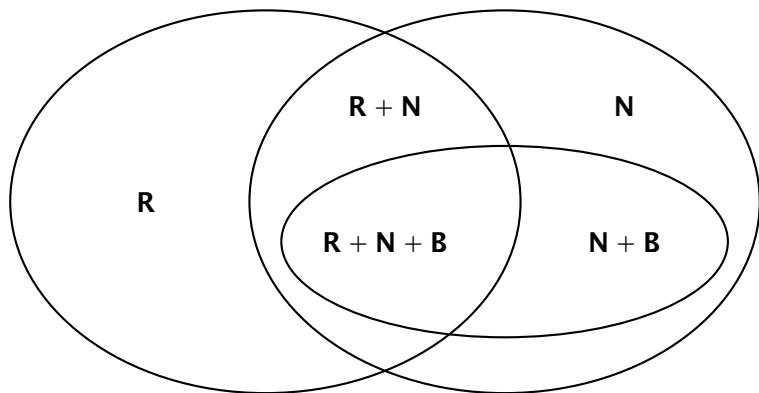
- no catalyzer can help starting the session on b

Definition

a is **boundable** if it is never followed by free channels

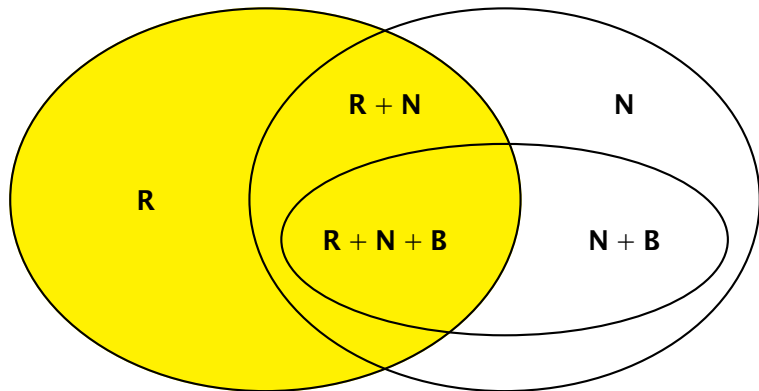
- b is nested but not boundable

Service classification



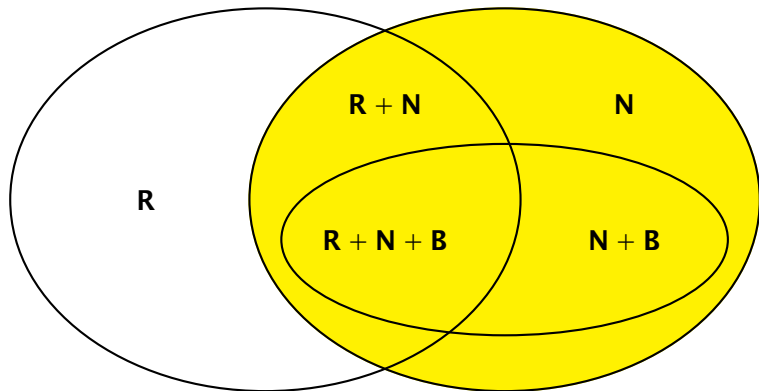
- each service can have up to three features ...
- ... which the interaction type system **guesses**

Service classification



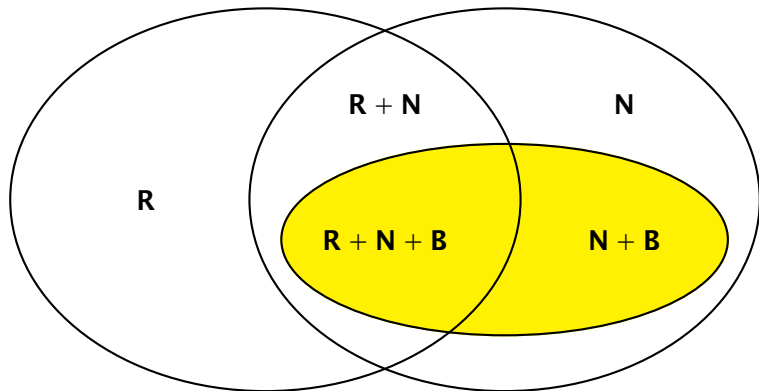
- each service can have up to three features ...
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Service classification



- each service can have up to three features ...
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Service classification



- each service can have up to three features ...
- ... which the interaction type system **guesses**

Algorithm judgments

$P \Rightarrow D; R; N; B$

If...

$$\begin{aligned} D^\infty &\subseteq N \setminus R \\ D \downarrow N &\subseteq N \\ fs(P) &\subseteq R \cup N \end{aligned}$$

Example 1

$a(y).b(z).y?(x).z!\langle x \rangle \Rightarrow$

Example 1

$$\frac{\frac{\frac{\mathbf{0} \Vdash}{\hline} z!\langle x \rangle \Vdash}{\hline} y?(x).z!\langle x \rangle \Vdash}{\hline} b(z).y?(x).z!\langle x \rangle \Vdash}{\hline} a(y).b(z).y?(x).z!\langle x \rangle \Vdash$$

Example 1

all services have all features

$0 \models \emptyset; S; S; S$

$z!\langle x \rangle \models$

$y?(x).z!\langle x \rangle \models$

$b(z).y?(x).z!\langle x \rangle \models$

$a(y).b(z).y?(x).z!\langle x \rangle \models$

Example 1

$$\frac{\frac{\frac{\mathbf{0} \models \emptyset; S; S; S}{z!\langle x \rangle \models \emptyset; S; S; S}}{y?(x).z!\langle x \rangle \models}}{b(z).y?(x).z!\langle x \rangle \models}}{a(y).b(z).y?(x).z!\langle x \rangle \models}$$

Example 1

$$\frac{\frac{\frac{\mathbf{0} \Rightarrow \emptyset; S; S; S}{z!\langle x \rangle \Rightarrow \emptyset; S; S; S}}{y?(x).z!\langle x \rangle \Rightarrow \{y < z\}; S; S; S}}{b(z).y?(x).z!\langle x \rangle \Rightarrow}}{a(y).b(z).y?(x).z!\langle x \rangle \Rightarrow}$$

Example 1

$$\frac{\frac{\frac{\mathbf{0} \Rightarrow \emptyset; S; S; S}{z!\langle x \rangle \Rightarrow \emptyset; S; S; S}}{y?(x).z!\langle x \rangle \Rightarrow \{y < z\}; S; S; S}}{b(z).y?(x).z!\langle x \rangle \Rightarrow \{y < b\}; S; S \setminus \{b\}; S \setminus \{b\}}}{a(y).b(z).y?(x).z!\langle x \rangle \Rightarrow}$$

$B \subseteq N$

$D \downarrow N \subseteq N$

Example 1

$$\frac{\frac{\frac{\mathbf{0} \Rightarrow \emptyset; S; S; S}{z!\langle x \rangle \Rightarrow \emptyset; S; S; S}}{y?(x).z!\langle x \rangle \Rightarrow \{y < z\}; S; S; S}}{b(z).y?(x).z!\langle x \rangle \Rightarrow \{y < b\}; S; S \setminus \{b\}; S \setminus \{b\}}}{a(y).b(z).y?(x).z!\langle x \rangle \Rightarrow \{a < b\}; S; S \setminus \{b\}; S \setminus \{b\}}$$

Example 1 (cont.)

$$\bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle$$

$$a(y) \cdots \mid \bar{a}(y) \cdots \Rightarrow$$

Example 1 (cont.)

$$\frac{\frac{\frac{\mathbf{0} \Rightarrow \emptyset; S; S; S}{y!\langle x \rangle \Rightarrow \emptyset; S; S; S}}{z?(x).y!\langle x \rangle \Rightarrow \{z < y\}; S; S; S}}{\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \{b < y\}; S; S; S}}{\bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \{b < a\}; S; S; S}$$

$$a(y) \cdots \mid \bar{a}(y) \cdots \Rightarrow$$

Example 1 (cont.)

$$\frac{\frac{\frac{\mathbf{0} \Vdash \emptyset; S; S; S}{y!\langle x \rangle \Vdash \emptyset; S; S; S}}{z?(x).y!\langle x \rangle \Vdash \{z < y\}; S; S; S}}{\bar{b}(z).z?(x).y!\langle x \rangle \Vdash \{b < y\}; S; S; S}}{\bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Vdash \{b < a\}; S; S; S}$$

$$a(y) \cdots \Vdash \{a < b\}; S; S \setminus \{b\}; S \setminus \{b\}$$

$$a(y) \cdots \mid \bar{a}(y) \cdots \Vdash$$

Example 1 (cont.)

$$\frac{\frac{\frac{\mathbf{0} \Vdash \emptyset; S; S; S}{y!\langle x \rangle \Vdash \emptyset; S; S; S}}{z?(x).y!\langle x \rangle \Vdash \{z < y\}; S; S; S}}{\bar{b}(z).z?(x).y!\langle x \rangle \Vdash \{b < y\}; S; S; S}}{\bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Vdash \{b < a\}; S; S; S}$$

$$\frac{a(y) \cdots \Vdash \{a < b\}; S; S \setminus \{b\}; S \setminus \{b\} \quad \bar{a}(y) \cdots \Vdash \{b < a\}; S; S; S}{a(y) \cdots \mid \bar{a}(y) \cdots \Vdash}$$

Example 1 (cont.)

$$\begin{array}{c}
 \mathbf{0} \Vdash \emptyset; S; S; S \\
 \hline
 y! \langle x \rangle \Vdash \emptyset; S; S; S \\
 \hline
 z?(x).y! \langle x \rangle \Vdash \{z < y\}; S; S; S \\
 \hline
 \bar{b}(z).z?(x).y! \langle x \rangle \Vdash \{b < y\}; S; S; S \\
 \hline
 \bar{a}(y).\bar{b}(z).z?(x).y! \langle x \rangle \Vdash \{b < a\}; S; S; S
 \end{array}$$

$$\begin{array}{c}
 a(y) \cdots \Vdash \{a < b\}; S; S \setminus \{b\}; S \setminus \{b\} \quad \bar{a}(y) \cdots \Vdash \{a < a\}; S; S; S \\
 \hline
 a(y) \cdots \mid \bar{a}(y) \cdots \Vdash \{a < b, b < a\}; S \setminus \{a, b\}; S \setminus \{b\}; S \setminus \{b\}
 \end{array}$$

$D^\infty \subseteq \mathbb{N} \setminus \mathbb{R}$

Example 1 (cont.)

$$\frac{\frac{\frac{\frac{\mathbf{0} \Vdash \emptyset; S; S; S}{y!\langle x \rangle \Vdash \emptyset; S; S; S}}{z?(x).y!\langle x \rangle \Vdash \{z < y\}; S; S; S}}{\bar{b}(z).z?(x).y!\langle x \rangle \Vdash \{b < y\}; S; S; S}}{\bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Vdash \{b < a\}; S; S; S}}$$

$$\frac{a(y) \cdots \Vdash \{a < b\}; S; S \setminus \{b\}; S \setminus \{b\} \quad \bar{a}(y) \cdots \Vdash \{b < a\}; S; S; S}{a(y) \cdots \mid \bar{a}(y) \cdots \Vdash \{a < b, b < a\}; S \setminus \{a, b\}; S \setminus \{b\}; S \setminus \{b\}}$$

Example 2

$$\bar{c}(t).t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \vdash$$

$$a(y) \cdots \vdash \{a < b\}; S; S \setminus \{b\}; S \setminus \{b\} \quad \bar{c}(t) \cdots \vdash \emptyset; S \setminus \{b\}; S; S$$

$$a(y) \cdots \mid \bar{c}(t) \cdots \vdash \{a < b\}; S \setminus \{b\}; S \setminus \{b\}; S \setminus \{b\}$$

Example 2

⋮

$$\frac{}{\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \{b < y\}; S; S; S}$$

$$\frac{}{\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow}$$

$$\frac{}{t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow}$$

$$\frac{}{\bar{c}(t).t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow}$$

$$\frac{a(y) \cdots \Rightarrow \{a < b\}; S; S \setminus \{b\}; S \setminus \{b\} \quad \bar{c}(t) \cdots \Rightarrow \emptyset; S \setminus \{b\}; S; S}{a(y) \cdots | \bar{c}(t) \cdots \Rightarrow \{a < b\}; S \setminus \{b\}; S \setminus \{b\}; S \setminus \{b\}}$$

Example 2

$$\begin{array}{c}
 \vdots \text{ x must be nested, so } b \text{ too} \\
 \hline
 \bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \{b < y\}; S; S; S \\
 \hline
 \bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \emptyset; S \setminus \{b\}; S; S \\
 \hline
 t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \text{dependencies discharged} \\
 \hline
 \bar{c}(t).t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \\
 \hline
 a(y) \cdots \Rightarrow \{a < b\}; S; S \setminus \{b\}; S \setminus \{b\} \quad \bar{c}(t) \cdots \Rightarrow \emptyset; S \setminus \{b\}; S; S \\
 \hline
 a(y) \cdots | \bar{c}(t) \cdots \Rightarrow \{a < b\}; S \setminus \{b\}; S \setminus \{b\}; S \setminus \{b\}
 \end{array}$$

Example 2

⋮

$$\frac{}{\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \{b < y\}; S; S; S}$$

$$\frac{}{\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \emptyset; S \setminus \{b\}; S; S}$$

$$\frac{}{t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \emptyset; S \setminus \{b\}; S; S}$$

$$\frac{}{\bar{c}(t).t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \emptyset; S \setminus \{b\}; S; S}$$

$$a(y) \cdots \Rightarrow \{a < b\}; S; S \setminus \{b\}; S \setminus \{b\} \quad \bar{c}(t) \cdots \Rightarrow \emptyset; S \setminus \{b\}; S; S$$

$$a(y) \cdots \mid \bar{c}(t) \cdots \Rightarrow \{a < b\}; S \setminus \{b\}; S \setminus \{b\}; S \setminus \{b\}$$

Example 2

⋮

$$\frac{}{\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \{b < y\}; S; S; S}$$

$$\frac{}{\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \emptyset; S \setminus \{b\}; S; S}$$

$$\frac{}{t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \emptyset; S \setminus \{b\}; S; S}$$

$$\frac{}{\bar{c}(t).t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \emptyset; S \setminus \{b\}; S; S}$$

$$a(y) \cdots \Rightarrow \{a < b\}; S; S \setminus \{b\}; S \setminus \{b\} \quad \bar{c}(t) \cdots \Rightarrow \emptyset; S \setminus \{b\}; S; S$$

$$a(y) \cdots \mid \bar{c}(t) \cdots \Rightarrow \{a < b\}; S \setminus \{b\}; S \setminus \{b\}; S \setminus \{b\}$$

Example 2

⋮

$$\frac{}{\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \{b < y\}; S; S; S}$$

$$\frac{}{\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \emptyset; S \setminus \{b\}; S; S}$$

$$\frac{}{t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \emptyset; S \setminus \{b\}; S; S}$$

$$\frac{}{\bar{c}(t).t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \emptyset; S \setminus \{b\}; S; S}$$

$$\frac{a(y) \cdots \Rightarrow \{a < b\}; S; S \setminus \{b\}; S \setminus \{b\} \quad \bar{c}(t) \cdots \Rightarrow \emptyset; S \setminus \{b\}; S; S}{a(y) \cdots \mid \bar{c}(t) \cdots \Rightarrow \{a < b\}; S \setminus \{b\}; S \setminus \{b\}; S \setminus \{b\}}$$

Result

Theorem

If $P \Vdash D; R; N; B$, then P has progress

Proof.

The algorithm is sound and complete wrt the inference type system (cf. CONCUR 2008) □

Result

Theorem

If $P \mapsto D; R; N; B$, then P has progress

Proof.

The algorithm is sound and complete wrt the inference type system (cf. CONCUR 2008) (for finite processes only) \square

Soon to come

Inference for recursive processes

Wrap up

- static analysis for (multiparty) session interleaving
- progress \neq absence of deadlock
 - diverging systems do not necessarily have progress
 - catalyzers may help reduction
- efficient inference algorithm

Future work

- many simple program patterns are **ill typed**
 - ☞ more flexible type discipline is required
- π -calculus \neq **programming language**
 - ☞ richer/more compositional types are needed
- type systems for liveness properties are **complex**
 - ☞ traditional concepts/techniques (fairness, subtyping, coinductive reasoning, ...) must be revisited

A simple ill-typed process with progress

$\text{def } X(y, z) = y!\langle 3 \rangle.z?(x).X(y, z) \text{ in } b \prec a$
 $\text{def } Y(y, z) = y?(x).z!\langle x \rangle.Y(y, z) \text{ in } a \prec b$
 $a(y).b(z).X(y, z) \mid \bar{a}(y).\bar{b}(z).Y(y, z)$